

Bound States of Type I' D-particles and Enhanced Gauge Symmetry

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Duality between the $E_8 \times E_8$ heterotic string and Type I' theory predicts a tower of D(irichlet)-particle bound states corresponding to perturbative heterotic string states. In the limit of infinite Type I' coupling, some of these bound states become massless, giving rise to enhanced $E_8 \times E_8$ gauge symmetry. By taking a different infinite coupling limit, one can recover the $E_8 \times E_8$ gauge bosons of M-theory, compactified on S^1/\mathbb{Z}_2 . In this paper we use the matrix model description of the D-particle dynamics to study these bound states. We find results consistent with the chain of dualities and clarify a number of issues that arise in the application of the matrix mechanics to this system.

1. Introduction

Recently there has been much study of the strong coupling behavior of ten-dimensional supersymmetric string theories, and their simplest compactifications. A complex web of dualities has emerged, unifying the ten-dimensional string theories and relating them to the eleven-dimensional theory known as M-theory. These strong/weak coupling dualities predict correspondences between the perturbative states of one string theory and nonperturbative bound states of another, allowing stringent tests of the proposed dualities to be made. In this paper we test Type I'/ $E_8 \times E_8$ heterotic duality, and Type I'/M-theory duality by studying the spectrum of bound states of D(irichlet)-particles in the Type I' theory.

The Type I' theory is T-dual to the usual Type I superstring theory compactified to nine dimensions, and may be constructed as an orientifold of Type IIA theory, which contains a background of 16 D8-branes [1]. The Type I theory in ten-dimensions is related via strong/weak coupling duality to the $SO(32)$ heterotic string [2,3]. The Type I' theory is therefore dual to the $SO(32)$ heterotic theory compactified to nine dimensions and because the two versions of the heterotic theory are related by T-duality below ten dimensions [4], the Type I' theory is related via strong/weak coupling duality to the $E_8 \times E_8$ heterotic theory. This duality predicts a tower of D-particle bound states which must match the perturbative spectrum of the $E_8 \times E_8$ heterotic theory [5] (suitably broken by Wilson lines).

The strongly coupled $E_8 \times E_8$ heterotic string has been conjectured to be dual to a compactification of M-theory on an interval S^1/\mathbb{Z}_2 [6]. This is related via a duality to a different strong coupling limit of Type I' theory. Again a tower of D-particle bound states are predicted that may be checked against the M-theory spectrum.

The D-particle bound states in Type IIA string theory are equivalent to supersymmetric ground states in the quantum mechanics of a matrix model studied some time ago in [7], and more recently in [8–11]. In these works much was learned about the spectrum of non-BPS states, and scattering of single-particle BPS states. The existence of supersymmetric bound states of two or more D-particles has not yet been established. In certain truncations of the matrix quantum mechanics describing two D-particles [12], it has been argued that a normalizable supersymmetric ground state does not exist. Later we will comment on why the bound states considered in the present paper do not survive the analog of the truncations used in [12], providing a possible explanation of these results.

D-particle bound states in Type I' theory were originally considered by Danielsson and Ferretti [13], where the matrix model describing their dynamics was constructed and non-BPS excitations and D-particle scattering were studied. The appearance of the BPS

bound states required by Type I'/heterotic and Type I'/M-theory¹ duality was motivated in [5].

In the present work, we continue the study of these bound states. The direct construction of such states in the matrix model formulation is in general a very difficult task. To make progress, we will use a Born-Oppenheimer approximation. This approach is similar to that of [13] but differs in some important details. The bound states that show up in this approximation provide constraints on the quantum numbers of the exact bound states. For low values of the D-particle number these states are explicitly constructed, and the quantum numbers found are consistent with the proposed dualities.

2. Predictions of Duality

As discussed in [5] duality between the Type I' theory and $E_8 \times E_8$ heterotic string theory compactified to 9 dimensions predicts the existence of certain bound states of D-particles and D8-branes. This may be seen by compactifying the $SO(32)$ heterotic string theory on a circle with Wilson line $(\frac{1}{2}^8, 0^8)$, in standard notation. This is related by T -duality to a compactification of the $E_8 \times E_8$ heterotic string [4]. The parameters of the two theories are related as

$$\lambda_E = \frac{R_{I'}^{3/2}}{\lambda_{I'}^{1/2}} , \quad R_E = \sqrt{R_{I'} \lambda_{I'}} . \quad (2.1)$$

The Wilson line breaks the gauge symmetry to $SO(16) \times SO(16)$. In the limit that $R_E \rightarrow \infty$ with λ_E fixed, states which become massless have even Kaluza-Klein momentum and give the $N = 1$ gravity multiplet with spacetime (i.e. $SO(8)$) quantum numbers $\mathbf{1} + \mathbf{28} + \mathbf{35}_v + \mathbf{8}_s + \mathbf{56}_s$, plus the adjoint $(\mathbf{1}, \mathbf{120}) + (\mathbf{120}, \mathbf{1})$ of $SO(16) \times SO(16)$ in the $\mathbf{8}_v$ and $\mathbf{8}_s$; or have odd Kaluza-Klein momentum and give spinor reps $(\mathbf{1}, \mathbf{128}) + (\mathbf{128}, \mathbf{1})$ in the $\mathbf{8}_v$ and $\mathbf{8}_s$. These multiplets combine to give $E_8 \times E_8$ gauge bosons and the ten-dimensional $N = 1$ supergravity multiplet in the decompactification limit. In terms of the Type I' string, the states with Kaluza-Klein momentum n map to bound states of n D-particles with the D8-branes. For n -even, we expect to find the $N = 1$ gravity multiplet, plus the $(\mathbf{1}, \mathbf{120}) + (\mathbf{120}, \mathbf{1})$ charged states; for n -odd we expect to find the $(\mathbf{1}, \mathbf{128}) + (\mathbf{128}, \mathbf{1})$ states.

¹ The M(atr ix)-theory [14] limit of the Type I' D-particle matrix model is obtained by sending the D-particle number to infinity. Following [14] this matrix model should provide a definition of quantum eleven-dimensional M-theory on S^1/\mathbb{Z}_2 .

Both these theories may be viewed as different compactifications of M-theory [6] on $S^1 \times S^1/\mathbb{Z}_2$, with radii R_0 and R_1 . The relations between the parameters of the theories are as follows

$$R_0 = \lambda_{I'}^{2/3} = \frac{R_E}{\lambda_E^{1/3}} , \quad R_1 = \frac{R_{I'}}{\lambda_{I'}^{1/3}} = \lambda_E^{2/3} . \quad (2.2)$$

One may also consider the limit R_0 and $R_1 \rightarrow \infty$, when the massless fields should become the usual eleven-dimensional supergravity multiplet propagating in the bulk, together with massless $E_8 \times E_8$ gauge bosons propagating on the ends of the interval. In terms of Type I' parameters we have $R_{I'} \rightarrow \infty$ and $\lambda_{I'} \rightarrow \infty$. Duality between Type I' and M-theory predicts bound states of even numbers of D-particles in the

$$(\mathbf{8}_v + \mathbf{8}_s) \times (\mathbf{8}_v + \mathbf{8}_c) = \mathbf{1} + \mathbf{28} + \mathbf{35}_v + \mathbf{8}_v + \mathbf{56}_v + \mathbf{8}_s + \mathbf{8}_c + \mathbf{56}_s + \mathbf{56}_c , \quad (2.3)$$

of $SO(8)$ will fill out the remainder of the eleven-dimensional supergravity multiplet. The modes carrying zero momentum in the S^1/\mathbb{Z}_2 direction will be the same as in the heterotic case, mentioned above. Bound states with $SO(16) \times SO(16)$ charge will give rise to $E_8 \times E_8$ gauge multiplets, which only propagate on the ends of the interval and lie in the $\mathbf{8}_v + \mathbf{8}_s$ as before.

3. Bound States of D-particles and D8-branes

Let us now consider these bound states directly in the Type I' formalism. Here we will clarify a number of points not addressed in [5]. The Hamiltonian governing the 0-8 brane system is discussed in [13]. This describes the situation where 8 D 8-branes are sitting at each orientifold plane. This system has $SO(16) \times SO(16)$ spacetime gauge symmetry, and is dual to the heterotic string compactification discussed above. Only in this limit is the Type I' string coupling a constant as a function of the compactified coordinate.

As discussed in [3], if we consider the situation when n 8-branes are at $x^9 = 0$ and $16 - n$ are at $x^9 = \pi R_{I'}$, the Type I' theory will develop a linear dilaton background, provided the radius is sufficiently large. There exists a critical radius at which the coupling of Type I' diverges at the end with the least number of 8-branes. In this limit the D-particle bound states will become massless, and should make up multiplets of $E_8 \times E_8$. For the situation at hand, the dilaton is constant and $E_8 \times E_8$ multiplets will appear in the infinite coupling limit for arbitrary values of the radius.

Let us review the $SO(N)$ symmetric matrix model [13] governing the low-energy dynamics of N D-particles near one of the orientifold planes. The bosonic worldline fields are as follows:

$$A_{0,9}^{IJ} , \quad X_i^{IJ} , \quad x_i^0 , \quad (3.1)$$

where A is in the adjoint of $SO(N)$, X is in the symmetric traceless rep and x_i^0 is a singlet representing the position of the center of mass of the D-particles. Here $I, J = 1, \dots, N$ are $SO(N)$ indices and $i = 1, \dots, 8$ runs over the 8 noncompact spatial dimensions. The fermionic modes are:

$$S_a^{IJ} , \quad S_{\dot{a}}^{IJ} , \quad s_{\dot{a}} , \quad (3.2)$$

where S_a is in the adjoint, $S_{\dot{a}}$ is in the traceless symmetric rep and $s_{\dot{a}}$ is a singlet. Here a and \dot{a} label the spinor reps $\mathbf{8}_s$ and $\mathbf{8}_c$ of the eight noncompact spatial dimensions. Finally there are the degrees of freedom arising from open strings starting at a D-particle and ending on one of the D8-branes, χ_r^I which live in the fundamental of $SO(N)$. Here $r = 1, \dots, 16$ labels the different 8-branes and their mirrors. The Hamiltonian is given by

$$\begin{aligned} H = & \text{Tr} \left(\lambda_{I'} \left(\frac{1}{2} P_i^2 - \frac{1}{2} E_9^2 \right) + \frac{1}{\lambda_{I'}} \left(\frac{1}{2} [A_9, X_i]^2 - \frac{1}{4} [X_i, X_j]^2 \right) \right. \\ & + \frac{i}{2} \left(-S_a [|A_9|, S_a] - S_{\dot{a}} [|A_9|, S_{\dot{a}}] + 2X_i \sigma_{a\dot{a}}^i \{S_a, S_{\dot{a}}\} \right) \\ & \left. + \frac{i}{2} (\chi_r^I |A_{9IJ}| \chi_r^J + \chi_r^I B_\mu^{rs} \chi_s^I) \right) , \end{aligned} \quad (3.3)$$

in a gauge where A_0 has been set to zero. The $\sigma_{a\dot{a}}^i$ are defined as in [13,10]. The B_μ comes from the coupling to the usual spacetime gauge bosons. Note that fixing gauge $A_1 = 0$ requires us to impose the Gauss law constraint on the space of physical states.

In fact, this Hamiltonian is not quite correct. So far, we have ignored open string winding modes which wrap the compactified direction (remember winding number is conserved in Type I' theory, while Kaluza-Klein momentum is not). The dynamics of such modes may be ignored when the distance between the orientifold planes is large. However, one effect of these modes is to introduce a normal-ordering constant multiplying the terms in the Hamiltonian linear in A . The first case in which this term is nontrivial is the two D-particle case. In the following we will compute this term and show it plays a crucial role in the construction of the bound states.

For zero D-particle number, the massless states correspond to the usual $N = 1$ supergravity multiplet in nine dimensions together with $SO(16) \times SO(16)$ gauge bosons arising from the usual $8 - 8$ open strings.

With just a single D-particle present the Hamiltonian is trivial and independent of the fields χ . The quantum numbers of the states arise from quantizing the fermion zero modes, as follows. Define linear combinations of the χ_r ,

$$b_j = \frac{1}{2} (\chi_j + i\chi_{j+8}) , \quad b_j^* = \frac{1}{2} (\chi_j - i\chi_{j+8}) , \quad (3.4)$$

which satisfy the anticommutation relations

$$\{b_i, b_j\} = 0, \quad \{b_i^*, b_j^*\} = 0, \quad \{b_i, b_j^*\} = \delta_{ij}. \quad (3.5)$$

The states $b_i^*|0\rangle$, $b_i^*b_j^*|0\rangle$ etc. give a 2^8 dim rep of $SO(16)$ which is the $\mathbf{128} + \mathbf{128}'$. In addition, one must project out by an analog of the heterotic GSO projection. In Type I' this appears as a discrete \mathbb{Z}_2 gauge symmetry, which is a remnant of the gauge field on the D-particle projected out by the orientifold projection. On the χ fields, this will act as $(-1)^F$, so to project onto gauge invariant states we keep only those made up of even numbers of χ 's – yielding a single $\mathbf{128}$ of $SO(16)$. The other worldline fields are invariant under the discrete gauge symmetry. The spacetime quantum numbers arise by quantizing the $s_{\hat{a}}$ zero modes. This is identical to the quantization of fermion zero modes in the Green-Schwarz formulation of the Type I string [15]. The result is that a $\mathbf{8}_v$ and $\mathbf{8}_s$ of $SO(8)$ appear. This is in accord with Type I'/heterotic and Type I'/M-theory duality.

We now consider bound states of a pair of D-particles. It is helpful to rewrite the general Hamiltonian (3.3) for this particular case. In terms of $SO(2)$ matrices, the A_9 , X_i , S_a and $S_{\hat{a}}$ fields are

$$\begin{aligned} A_9^{IJ} &= \frac{i}{2}\sigma_2 A \\ X_i^{IJ} &= \frac{1}{2}(x_i\sigma_1 + \tilde{x}_i\sigma_3) \\ S_a^{IJ} &= \frac{i}{2}\sigma_2 S_a \\ S_{\hat{a}}^{IJ} &= \frac{1}{2}(S_{\hat{a}}\sigma_1 + \tilde{S}_{\hat{a}}\sigma_3), \end{aligned} \quad (3.6)$$

where the σ_i^{IJ} are the usual Pauli matrices. The χ fields live in the doublet of $SO(2)$, i.e. $\chi = (\chi^1, \chi^2)$. Substituting this into (3.3) we find

$$\begin{aligned} H &= \frac{1}{4}\lambda_{I'}(p_i^2 + \tilde{p}_i^2 + p_A^2) + \frac{1}{\lambda_{I'}}\left(\frac{1}{8}(x_i\tilde{x}_j - \tilde{x}_ix_j)^2 + \frac{1}{4}A^2(x_i^2 + \tilde{x}_i^2)\right) \\ &+ \frac{i}{2}(-\tilde{S}_{\hat{a}}|A|S_{\hat{a}} + \sigma_{a\hat{a}}^i S_a(S_{\hat{a}}\tilde{x}_i - \tilde{S}_{\hat{a}}x_i)) + \frac{i}{4}\chi^1|A|\chi^2. \end{aligned} \quad (3.7)$$

The nontrivial (anti)commutation relations for the different fields are:

$$\begin{aligned} i[p_i, x_j] &= \delta_{ij}, & i[\tilde{p}_i, \tilde{x}_j] &= \delta_{ij}, & i[p_A, A] &= 1, & \{S_a, S_b\} &= \delta_{ab}, \\ \{S_{\hat{a}}, S_{\hat{b}}\} &= \delta_{\hat{a}\hat{b}}, & \{\tilde{S}_{\hat{a}}, \tilde{S}_{\hat{b}}\} &= \delta_{\hat{a}\hat{b}}, & \{\chi_r^i, \chi_s^j\} &= \delta^{ij}\delta_{rs}. \end{aligned} \quad (3.8)$$

To proceed further it is convenient to make a Born-Oppenheimer approximation to study the two D-particle bound states. The Hamiltonian only gives a correct description of

D-particle dynamics in the limit that their separation is smaller than l_s , the string length. We may treat A as a slowly fluctuating mode and consider a limit in which the length scale set by A is less than l_s , but much greater than the eleven-dimensional Planck scale l_{pl} . This essentially corresponds to the moduli space approximation, and was discussed in the Type II context in [11]. The approximation breaks down when one considers ground states. The wavefunctions of these states may depend sensitively on the $|A| < l_{pl}$ region. However, because we know the wavefunction varies in a smooth way as $|A|$ increases, the exact bound state wavefunctions should match onto the approximate Born-Oppenheimer wavefunctions for $|A|$ sufficiently large. This puts strong constraints on the possible quantum numbers of the exact bound states, and we will find it completely determines the $SO(16)$ gauge charges.

The Born-Oppenheimer approximation proceeds as follows. The degrees of freedom are separated into the fast modes for which the wavefunction is expected to vary rapidly, and the slow modes for which the opposite is true. For fixed values of the slow modes (which we take here to be A , S_a and the singlets x_i^0 and $s_{\hat{a}}$) we solve for the eigenfunctions of the fast modes. In this limit the x_i and \tilde{x}_i may be treated as ordinary harmonic oscillators (the quartic piece in the x 's becomes irrelevant). Likewise the fermionic fields $S_{\hat{a}}$, $\tilde{S}_{\hat{a}}$ and the χ_r should be treated as the Grassman analog of the harmonic oscillator.

As mentioned above, there is an additional contribution to the Hamiltonian coming from open string winding states. At low energies these modes are in their ground states, but they may nevertheless introduce extra normal-ordering terms into the Hamiltonian. For us, the interesting modes are those which start on one D-particle, wrap n times around the S^1 and end on a 8-brane or the mirror D-particle. This introduces an extra piece linear in A , which may be computed as follows. First, let us define

$$\begin{aligned}
\chi_r &= \frac{1}{\sqrt{2}}(\chi_r^1 + i\chi_r^2) , & \bar{\chi}_r &= \frac{1}{\sqrt{2}}(\chi_r^1 - i\chi_r^2) , \\
\Sigma_{\hat{a}} &= \frac{1}{\sqrt{2}}(S_{\hat{a}} + i\tilde{S}_{\hat{a}}) , & \bar{\Sigma}_{\hat{a}} &= \frac{1}{\sqrt{2}}(S_{\hat{a}} - i\tilde{S}_{\hat{a}}) , \\
\alpha_k &= \frac{1}{2}\sqrt{\frac{A}{\lambda_{I'}}}(x_k + i\tilde{x}_k) + \frac{i}{2}\sqrt{\frac{\lambda_{I'}}{A}}(p_k + i\tilde{p}_k) , \\
\bar{\alpha}_k &= \frac{1}{2}\sqrt{\frac{A}{\lambda_{I'}}}(x_k - i\tilde{x}_k) + \frac{i}{2}\sqrt{\frac{\lambda_{I'}}{A}}(p_k - i\tilde{p}_k) ,
\end{aligned} \tag{3.9}$$

which allows us to write the terms in the Hamiltonian linear in A as

$$\delta H = \frac{1}{2}|A|(\alpha_k^\dagger \alpha_k + \bar{\alpha}_k^\dagger \bar{\alpha}_k + \bar{\Sigma}_{\hat{a}} \Sigma_{\hat{a}} + \frac{1}{2}\bar{\chi}_r \chi_r + c) . \tag{3.10}$$

The determination of the constant term c is reminiscent of the determination of the normal-ordering constants in the physical state conditions of heterotic string theory. First we need to consider more carefully the coupling of the winding modes to the gauge field A . The coupling is most simply determined using the T-dual formulation of the D-string in Type I [3]. All the winding modes are periodic around the compact direction except the winding modes of the χ fields, which are antiperiodic.² This implies the coupling of the winding modes to the A field takes the form

$$\delta H_{winding} = \sum_{n=1}^{\infty} \frac{1}{2} |A| \left(\alpha_{k,-n} \alpha_{k,n} + \bar{\alpha}_{k,-n} \bar{\alpha}_{k,n} + n \Sigma_{\dot{a},-n} \Sigma_{\dot{a},n} + \left(n - \frac{1}{2}\right) \bar{\chi}_{r,-n+1/2} \chi_{r,n-1/2} \right), \quad (3.11)$$

where n is the winding number and these modes obey the conventional (anti)commutation relations [15]. Thus we find that $\delta H_{winding}$ is precisely proportional to $L_0 + \bar{L}_0$ of the Green-Schwarz formulation of the heterotic string and the normal-ordering constant is computed in the same way. Identifying the $n = 1$ modes with the fields in (3.10) in the obvious way, and putting the rest of the winding modes in their ground states, we recover (3.10) with $c = -1$.

The Gauss law constraint is likewise analogous to the level-matching condition of the heterotic string – the operator

$$C = \alpha_k^\dagger \alpha_k - \bar{\alpha}_k^\dagger \bar{\alpha}_k - \bar{\Sigma}_{\dot{a}} \Sigma_{\dot{a}} + \frac{1}{2} \bar{\chi}_r \chi_r - 1, \quad (3.12)$$

should annihilate physical states.

The lowest energy eigenstates of the Hamiltonian for the fast modes, subject to this constraint are

$$|\psi\rangle = \bar{\chi}_r \bar{\chi}_s \prod_a S_a |0\rangle, \quad \text{and } \alpha_k^\dagger \prod_a S_a |0\rangle, \quad (3.13)$$

where $|0\rangle$ represents the ground state for all the bosonic modes. Note these states are even in the χ_r so are invariant under the \mathbb{Z}_2 discrete gauge symmetry.

To complete the Born-Oppenheimer approximation, we now quantize the slow modes. To do this we identify $H_{slow} = \langle \psi | H | \psi \rangle$. However this simply leaves us with $H_{slow} = \frac{1}{2} \lambda_{I'} p_A^2$. The lowest energy eigenfunction of this has wavefunction independent of A . This state is normalizable since A runs over the interval $[0, 2\pi R_{I'}]$. There are also other low-lying states with wavefunctions

$$\psi_{slow}(A) = \frac{1}{\sqrt{\pi R_{I'}}} \cos\left(\frac{pA}{R_{I'}}\right), \quad (3.14)$$

² There is a different sector of the theory where the winding modes of the χ fields are periodic – the analog of the NS sector of the D-string. This sector decouples from the low-energy physics.

with p an integer, corresponding to momentum in the 9 direction. Finally we must consider the fermion zero modes $s_{\dot{a}}$ and S_a . The $s_{\dot{a}}$ behave as for the single D-particle yielding spacetime quantum numbers $\mathbf{8}_v + \mathbf{8}_s$. In a similar way, the quantization of the S_a modes gives $\mathbf{8}_v + \mathbf{8}_c$. The final eigenstates are then a tensor product of (3.13) with the fermion zero mode states, yielding an adjoint **(120)** of $SO(16)$ with spacetime quantum numbers

$$(\mathbf{8}_v + \mathbf{8}_c) \times (\mathbf{8}_v + \mathbf{8}_s) = \mathbf{1} + \mathbf{28} + \mathbf{35}_v + \mathbf{8}_v + \mathbf{56}_v + \mathbf{8}_s + \mathbf{8}_c + \mathbf{56}_s + \mathbf{56}_c , \quad (3.15)$$

and a gauge singlet multiplet with spacetime quantum numbers

$$\begin{aligned} \mathbf{8}_v \times (\mathbf{8}_v + \mathbf{8}_s) \times (\mathbf{8}_v + \mathbf{8}_c) = & \mathbf{1} + \mathbf{8}_v + \mathbf{8}'_v + \mathbf{8}''_v + \mathbf{8}_s + \mathbf{8}'_s + \mathbf{8}_c + \mathbf{8}'_c + \mathbf{28} + \mathbf{28}' \\ & + \mathbf{35}_v + \mathbf{35}_s + \mathbf{35}_c + \mathbf{56}_v + \mathbf{56}_s + \mathbf{56}'_s + \mathbf{56}_c + \mathbf{56}'_c + \mathbf{112}_v \\ & + \mathbf{160}_v + \mathbf{160}'_v + \mathbf{160}_s + \mathbf{160}_c + \mathbf{224}_{vs} + \mathbf{224}_{vc} + \mathbf{350} . \end{aligned} \quad (3.16)$$

The states we have found include all the states predicted by Type I'/heterotic duality and Type I'/M-theory duality. We have also found additional bound states. However we expect the degeneracy arising from the S_a component of the wavefunctions will be split when we go to next order in the Born-Oppenheimer approximation due to the presence of the $XS_aS_{\dot{a}}$ term in the Hamiltonian. The spacetime quantum numbers of the exact bound states may therefore be a subset of those appearing on the right-hand side of (3.15) and (3.16). However, the gauge charges are unaffected by this splitting since the S_a are singlets of $SO(16)$.

In an improved approximation, we expect the states carrying gauge charge will have wavefunctions peaked near $A = 0$ with a width of order l_{pl} , and the states with $p \neq 0$ will become massive. On the other hand, since gravitons are free to propagate away from the orientifold plane, the whole tower of gauge singlet states with $p \neq 0$ should become light in the M-theory limit. Ideally, one would like to go beyond the Born-Oppenheimer approximation and compute, for example, the Witten index, which counts the number of normalizable zero-energy states with $(-1)^F$ weighting. Work in this direction is in progress [16].

We can see at this point how crucial it was that no truncations of the supersymmetric quantum mechanics were made. The zero-energy constraint and the Gauss law constraint take a form very similar to the physical state conditions of the heterotic string, which are sensitive to the spacetime dimension. This may explain the results of [12] where no bound state was found in the Type IIA context, when a truncation to four spacetime dimensions was considered. Of course here it has also been necessary to include an extra term in the Hamiltonian arising from normal-ordering terms coupling to winding states—

such terms will not be present in the Type II version of this calculation due to additional supersymmetry.

One could also have considered different Born-Oppenheimer approximations in which other combinations of the modes are treated as slow. Apparently, the zero-point energy from the fast modes in these other approximations is positive (barring any unexpected additional terms in the Hamiltonian). This is not the case for the Type II version of these calculations [12,17,18] where additional supersymmetry leads to cancellation of the zero-point energies along non-compact flat directions giving rise to a continuous spectrum, and many associated subtleties.

It should be pointed out that the $0+1$ -dimensional analog of the jumping phenomena found in higher dimensional theories [19] may occur as the coupling $\lambda_{I'}$ and the radius $R_{I'}$ are varied. In the preceding analysis we have assumed this does not occur, and we found results consistent with duality. It would be interesting to study this issue further to see whether the jumping conditions on the heterotic side correspond to those on the Type I' side.

Acknowledgments

We wish to thank J. Park, J. Polchinski and J. Schwarz for helpful discussions. This work was supported in part by DOE grant DE-FG03-92-ER40701.

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